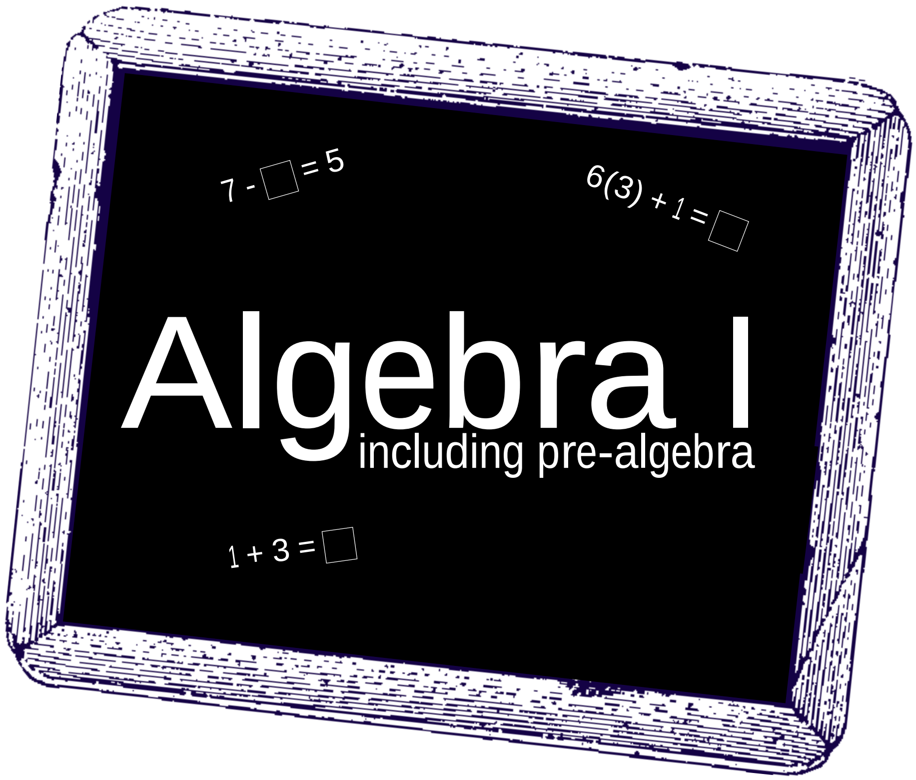
PRINCIPLES from PATTERNS



by David Quine

The Cornerstone Curriculum Project is the family ministry of David and Shirley Quine. We are dedicated to providing the best quality products at a reasonable price. Our ability to do so is dependent on your support. Principles from Patterns: Algebra is a consumable book, that is, the student pages are to be written on by your child. Please do not reproduce the student pages. You may purchase additional copies of this book for other children in your family from your local supplier or directly from The Cornerstone Curriculum Project. If you see the Student Book at a used curriculum sale, please do not buy it.

The Cornerstone Curriculum Project
2006 Flat Creek Place
Richardson, Texas 75080
(972) 235-5149

Table of Contents

UNIT 1: AN INTRODUCTION

UNIT 2: LINEAR EQUATIONS & LINEAR INEQUALITIES

UNIT 3: POLYNOMIAL EQUATIONS

UNIT 4: RATIONAL & RADICAL EQUATIONS

ACTIVITY 3 — 4
PRINCIPLE NUMBER 4

What Happens If …

You add two numbers in different positions?

EXPLORING THE CONCEPT —

Solve each side of the equation. Then compare the solutions to each side of the equation. Fill in the box with = of ≠.

1. 5 + 8 ☐ 8 + 5

2. 12 + 71 ☐ 71 + 12

3. 31 + 19 ☐ 19 + 31

4. 10 + 20 ☐ 20 + 10

5. 24 + 9 ☐ 9 + 24

You select two numbers. Put them in the equation from left to right and then reverse the order. Fill in the box with =, or ≠.

6. ___ + ___ ☐ ___ + ___

7. ___ + ___ ☐ ___ + ___

8. ___ + ___ ☐ ___ + ___

9. ___ + ___ ☐ ___ + ___

10. ___ + ___ ☐ ___ + ___

---

11. Compare the numbers on each side of the box.
Are the numbers the same?

12. Are they in the same order or the reverse order?

13. Compare the symbol to each of the problems.
Are they equal or not equal?

14. What happens if you add numbers in one order and then in the reverse order?

    Using the letters 'x' and 'y' to represent any numbers,
    write an equation to represent the above pattern:

NAMING THE CONCEPT —

Two numbers can be added together in any position and the solution is still the same.
The solution is the same no matter how the numbers are positioned.
This is called the

COMMUTATIVE PRINCIPLE for ADDITION.

$$x + y = y + x$$

EXPANDING THE CONCEPT —

Is there a similar relationship involving multiplication?

What Happens If …

You change the position of the numbers to be multiplied?

Solve each side of the equation. Then compare the solutions to each side of the equation. Fill in the box with = of ≠.

1. 9 • 3 ☐ 3 • 9
2. 5 • 6 ☐ 6 • 5
3. 2 • 12 ☐ 12 • 2
4. 8 • 4 ☐ 4 • 8
5. 1 • 7 ☐ 7 • 1

6. Compare the numbers on each side of the box.
Are the numbers the same?

7. Are they in the same order or the reverse order?

8. Compare the symbols to each of the problems.
Are they equal or not equal?

9. What happens if you multiply numbers in different positions?

10. What would be a good name for this principle?

Using the letters 'x' and 'y' to represent any numbers, write an equation to represent the above pattern:

Two numbers can be multiplied together in any position and the solution is still the same.
The solution is the same no matter how the numbers are positioned.
This is called the

COMMUTATIVE PRINCIPLE for MULTIPLICATION.

$$x \cdot y = y \cdot x$$

1. What happens when you add or multiply numbers in different positions?

____

____

____

— COMMUTATIVE PRINCIPLE —

FUNDAMENTAL PRINCIPLE NUMBER 4

2. What is the COMMUTATIVE principle for ADDITION and MULTIPLICATION?

____

____

____

Does this principle apply to division? Yes No

3. $9 \div 3$ ☐ $3 \div 9$

4. $15 \div 3$ ☐ $3 \div 15$

5. $2 \div 12$ ☐ $12 \div 2$

6. $8 \div 4$ ☐ $4 \div 8$

ACTIVITY 3 — 5
PRINCIPLE NUMBER 5

EXPLORING THE CONCEPT —

What Happens If …

You multiply a number by the sum of two numbers or
multiply the number by the individual numbers and then add them together?

Solve each side of the equation. Solve the part in the ( ) first. Then compare the solutions to each side of the equation.
Fill in the box with = or ≠.

1. $5(6 + 7)$ ☐ $5 \cdot 6 + 5 \cdot 7$

2. $2(8 + 3)$ ☐ $2 \cdot 8 + 2 \cdot 3$

3. $7(2 + 4)$ ☐ $7 \cdot 2 + 7 \cdot 4$

4. $3(1 + 6)$ ☐ $3 \cdot 1 + 3 \cdot 6$

5. $9(5 + 2)$ ☐ $9 \cdot 5 + 9 \cdot 2$

6. Is the left side equal to the right side?

7. What happens if you multiply a number by the sum of two numbers or by the individual numbers?

Using the letters 'x', 'y', and 'z' to represent any numbers, write an equation to represent the above pattern:

NAMING THE CONCEPT —

A number multiplied by the sum of two numbers is the same as multiplying the number by the individual numbers. The solution is the same no matter if we add before we multiply or add after we multiply. This means that

MULTIPLICATION DISTRIBUTES OVER ADDITION.

$$x(y + z) = xy + xz$$

EXPANDING THE CONCEPT —

Is there a similar relationship involving subtraction?

What Happens If …

You multiply after you find the difference between two numbers or multiply and then find the difference?

Solve each side of the equation. Solve the part in the ( ) first. Then compare the solutions to each side of the equation.
Fill in the box with = of ≠.

1. 9(8 - 7) ☐ 9 • 8 - 9 • 7

2. 3(2 - 1) ☐ 3 • 2 - 3 • 1

3. 5(6 - 2) ☐ 5 • 6 - 5 • 2

4. 7(11 - 6) ☐ 7 • 11 - 7 • 6

5. 9(5 - 2) ☐ 9 • 5 - 9 • 2

6. Is the left side equal to the right side?

7. What happens if you multiply a number by the difference of two numbers or by the individual numbers and then find the difference?

8. What would be a good name for this principle?

Using the letters 'x', 'y', and 'z' to represent any numbers, write an equation to represent the above pattern:

A number multiplied by the difference between two numbers is the same as the number multiplied by each of the numbers in the difference and then subtracted. The solution is the same no matter if the subtraction is done before the multiplication or after the multiplication. It is said that

MULTIPLICATION DISTRIBUTES OVER SUBTRACTION.

$$x(y - z) = xy - xz$$

1. What happens when the addition or subtraction is done before the multiplication or you multiply numbers and then find the sum or difference?

— DISTRIBUTIVE PRINCIPLE —

FUNDAMENTAL PRINCIPLE NUMBER 5

2. What is the DISTRIBUTIVE principle for ADDITION and SUBTRACTION?

UNIT 1: An Introduction

Chapter 1: MATH SENTENCES
Chapter 2: A NEW NUMBER
Chapter 3: SEVEN PRINCIPLES
Chapter 4: EQUATIONS WITH ONE VARIABLE

UNIT 2: Linear Equations & Linear Inequalities

Chapter 5: EQUATIONS WITH TWO VARIABLES

UNIT 3: Polynomial Equations

Chapter 6: POLYNOMIALS
Chapter 7: THE QUADRATIC FORMULA
Chapter 8: FACTORING

UNIT 4: Rational & Radical Equations

Chapter 9: FRACTIONS INVOLVING VARIABLES
Chapter 10: THE PYTHAGOREAN PRINCIPLE

ACTIVITY 5 — 1

EQUATIONS WITH TWO VARIABLES

EXPLORING THE CONCEPT — Part A

Using unifix cubes make the following groupings. Write the corresponding math equation. After each problem join the cubes together to find the total and set them to the side.

GROUPING	EQUATION
1. Two groups of two with one left over.	2(2) + 1 = ☐
2. Two groups of three with one left over.	2(____) + ____ = ☐
3. Two groups of four with one left over.	____ (____) + ____ = ☐

Circle the number in each equation that you changed.

Was there a corresponding change in the solution?

Set the cubes side by side from left to right in the order in which they were made.

4. What pattern do you see?

5. Predict the next total number of cubes in the pattern.

6. Make two groups of five with one left over. ____ (____) + ____ = ____

7. Join these together and set them to the right of the other cubes. Was your prediction correct?

8. You increased the number in the ( ) by one.

What affect did this have on the total number of cubes?

9. What is the pattern?

Use the cubes and the graph paper labeled “Changing the number in front of the ( )” to make a picture of the pattern.

You will notice that there are two dark lines — one horizontal and the other vertical.
The horizontal line is labeled: Number in Each Group.
The vertical line is labeled: Total Number of Cubes.

Since the first grouping was two groups with two in each group with one left over, color 5 squares on the second vertical space. The next grouping was two groups with three in each group with one left over. Color 7 squares on the third vertical space. Do the same for the next two groupings.

10. What pattern do you see?

11. For every two spaces that you go up how many do you move to the right?

Draw a dot on the line at each point where the line begins to go up.

Now take a ruler and connect the dots.

12. Describe your picture now.

What did you draw … a curved line? … a straight line? … a jagged line?

13. What are the characteristics of this line?

14. For every two spaces you move up, how many do you move to the right?

15. Extend the line in both directions. Where does this line cross the major vertical line?

Since two numbers in the equation change we will use the letter x to represent the number in each group and the letter y to represent the total number of cubes. Add these two letters to the graph paper at the appropriate place.

Since numbers that change are called variables,
we can represent this straight line with this math equation:

$$2(x) + 1 = y \text{ or it may be re-written } y = 2x + 1$$

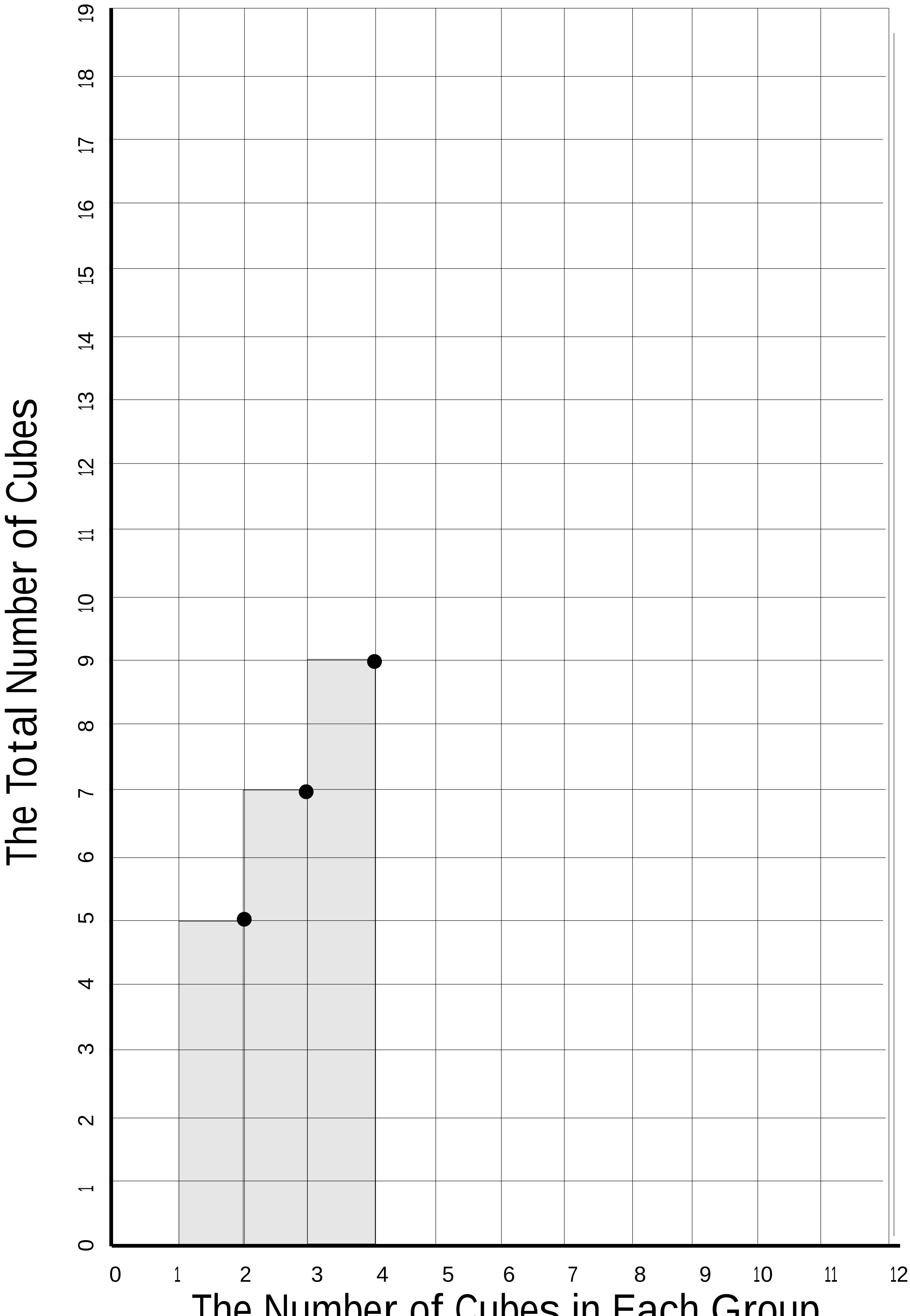
In the equation $y = 2x + 1$
IF the value of x is 0 … THEN $y = 2(0) + 1$. What would the value for y be? __________

IF the value of x is 1 … THEN $y = 2(1) + 1$. What would the value for y be? __________

IF the value of x is 2 … THEN $y = 2(2) + 1$. What would the value for y be? __________

IF the value of x is 3 … THEN $y = 2(3) + 1$. What would the value for y be? __________

IF the value of x is 4 … THEN $y = 2(4) + 1$. What would the value for y be? __________



The Total Number of Cubes
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
0
1
2
3
4
5
6
7
8
9
10
11
12
The Number of Cubes in Each Group

What Happens If …

The number in front of the ( ) is increased?
The number after the + symbol is increased?

Let's test your predictions. Begin by changing the number in front of the ( ).

Make the following groupings and write the corresponding equations.

GROUPING	EQUATION
17. Three groups of two with one left over.	3(2) + 1 = ☐
18. Three groups of three with one left over.	3 (_____) + _____ = _____
19. Three groups of four with one left over.	3 (_____) + _____ = _____

Set the cubes side by side from left to right in the order in which they were made.

20. What pattern do you see?

21. Predict the next solution in the pattern.

22. Make three groups of five with one left over. 3 (_____) + _____ = _____

23. Join these together and set them to the right of the other cubes. Was your prediction correct?

24. What is the pattern?

Use the cubes and the same graph paper as before to make a picture of the pattern.

Since the first grouping was three groups with two in each group with one left over, set the cubes on the second vertical space. The next grouping was three groups with three in each group with one left over. Set these cubes on the third vertical space. Do the same for the next two groupings. Draw around the edge of the cubes. Then remove the cubes.

25. What pattern do you see?

26. For every space that you move to the right how many do you move up?

Draw a dot on the line at each point where the line begins to go up.

Now take a ruler and connect the dots.

27. Describe your picture now.

What did you draw … a curved line? … a straight line? … a jagged line?

28. Extend the line in both directions. Where does this line cross the major vertical line?

29. What are the characteristics of this line?

… How is this line like the first?

…How is this line different from the first?

30. For every three spaces you move up, how many do you move to the right?

31. Using the letters x and y, write an equation to describe this line:

3 (_____) + 1 = _____

32. What affect does increasing the number in front of the ( ) have on

… the slope or slant of the line?

… the point where the line crosses the vertical y line.

Now let's test your prediction by changing the number one more time.
This time we will increase the number to 4.

Make the following groupings and write the corresponding equations.

GROUPING	EQUATION
33. Four groups of two with one left over.	4(2) + 1 = ☐
34. Four groups of three with one left over.	4 (_____) + _____ = _____
35. Four groups of four with one left over.	4 (_____) + _____ = _____

Set the cubes side by side from left to right in the order in which they were made.

36. What pattern do you see?

37. Predict the next solution in the pattern.

38. Make Four groups of five with one left over. 4 (_____) + _____ = _____

39. Join these together and set them to the right of the other cubes. Was your prediction correct?

40. What is the pattern?

Use the cubes and the graph paper as before.
Draw around the edge of the cubes. Then remove the cubes.

41. What pattern do you see?

42. For every four spaces that you go up how many do you move to the right?

Draw a dot on the line at each point where the line begins to go up.

Now take a ruler and connect the dots.

43. Describe your line now.

What did you draw … a curved line? … a straight line? … a jagged line?

44. Extend the line in both directions. Where does this line cross the major vertical line?

45. What are the characteristics of this line?

… How is this line like the first and second?

…How is this line different from the first and second?

46. For every space you move to the right, how many do you move up?

47. Using the letters x and y, write an equation to describe this line:

$$4(\_\_\_\_) + 1 = \_\_\_\_$$

48. What affect does increasing the number in front of the ( ) have on

…the slope or slant of the line?

… the point where the line crosses the vertical y line.

EXPLORING THE CONCEPT — Part B

What Happens If …

The number after the + symbol is increased?

Using unifix cubes make the following groupings.
Write the corresponding math equation.
After each problem join the cubes together to find the total and set them to the side.
Use the graph paper labeled "Changing the Number After the + Symbol."
Draw the line for the equation 2(x) + 1 = y (from Part A). This line will serve as a reference.

Now let's explore what happens if the PLUS 1 in the equation is changed to PLUS 2.

GROUPING	EQUATION
1. Two groups of two with two left over.	2(2) + 2 = ☐
2. Two groups of three with two left over.	2(3) + 2 = ☐
3. Two groups of four with two left over.	____ (____) + ____ = ☐

Set the cubes side by side from left to right in the order in which they were made.

4. What pattern do you see?

5. Predict the next total number of cubes in the pattern.

6. Make two groups of five with two left over. ____ (____) + ____ = ____

7. Join these together and set them to the right of the other cubes. Was your prediction correct?

Use the cubes and the graph paper to make a picture of the pattern. Since the first grouping was two groups with two in each group with two left over, color 6 squares on the second vertical space. The next grouping was two groups with three in each group with two left over. Color 8 squares on the third vertical space. Do the same for the next two groupings.

8. What pattern do you see?

9. For every space that you move to the right how many do you move up?

Draw a dot on the line at each point where the line begins to go up.

Now take a ruler and connect the dots.

10. Describe your picture now.

11. What did you draw … a curved line? … a straight line? … a jagged line?

12. Extend the line in both directions. Where does this line cross the major vertical line?

13. What are the characteristics of this line?

… How is this line like the first?

…How is this line different from the first?

14. For every two spaces you move up, how many do you move to the right?

15. Using the letters x and y, write an equation to describe this line:

2 (_____) + 2 = _____

16. What affect does increasing the number after the + symbol have on

…the slope or slant of the line?

… the point where the line crosses the vertical y line.

Now let's test your prediction by changing the number one more time.
This time we will increase the number to 4.

17. What affect will increasing the number after the + symbol to 4 have on

…the slope or slant of the line?

… the point where the line crosses the vertical y line?

… where do you think the line will cross the vertical y line?

Make the following groupings and write the corresponding equations.

GROUPING	EQUATION
18. Two groups of two with four left over.	2(2) + 4 = ☐
19. Two groups of three with four left over.	2 (_____) + _____ = _____
20. Two groups of four with four left over.	2 (_____) + _____ = _____

Set the cubes side by side from left to right in the order in which they were made.

21. What pattern do you see?

22. Predict the next solution in the pattern.

23. Make two groups of five with four left over. 2 (_____) + _____ = _____

Join these together and set them to the right of the other cubes.

24. Was your prediction correct?

25. What is the pattern?

Use the cubes and the graph paper as before. Make a bar graph of this equation.

26. What pattern do you see?

27. For every two spaces that you go up how many do you move to the right?

Draw a dot on the line at each point where the line begins to go up.

Now take a ruler and connect the dots.

28. Describe your line now.

What did you draw … a curved line? … a straight line? … a jagged line?

29. Extend the line in both directions. Where does this line cross the major vertical y line?

30. What are the characteristics of this line?

… How is this line like $2x + 1 = y$ and $2x + 2 = y$?

…How is this line different?

31. For every two spaces you move up, how many do you move to the right?

32. Using the letters x and y, write an equation to describe this line:

2 (_____) + 4 = _____

33. What affect does increasing the number after the + symbol have on

…the slope or slant of the line?

… the point where the line crosses the vertical y line.

32. To change the slant or slope of the line what part of the equation would you change?

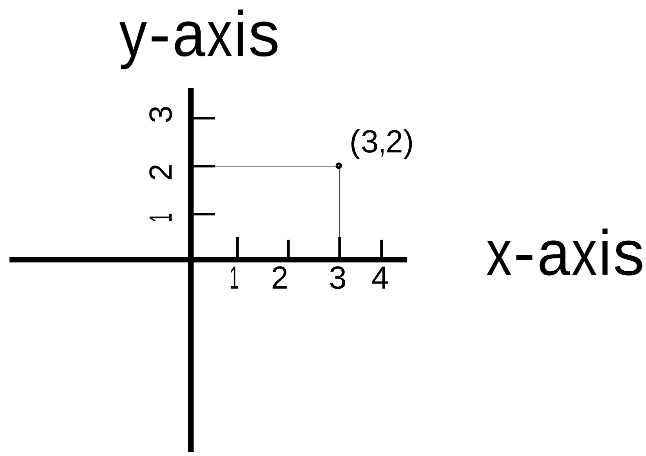
33. To change the point where the line crosses the vertical y line,
what part of the equation would you change?

34. These equations can all be pictured by a line.
What would be a good name for this kind of an equation?

NAMING THE CONCEPT

We have explored many different concepts in this activity.

The dark horizontal line of the graph is called the x-axis.
The dark vertical line of the graph is called the y-axis.



The point where the lines intersect is called the COORDINATES of the point. For example, the point where x = 3 and y = 2 intersect, written (3, 2), is found by moving across 3 units in the x-direction from the origin and moving up 2 units in the y-direction. This symbol (3,2) is called the ORDERED PAIR of the coordinates. The first number of the ordered pair is the x-value and is called the AB-SCISSA. The second number in the ordered pair is the y-value and is called the ORDINATE. The whole system — x-axis, y-axis, and all the points — is called the CARTESIAN COORDINATE SYSTEM.

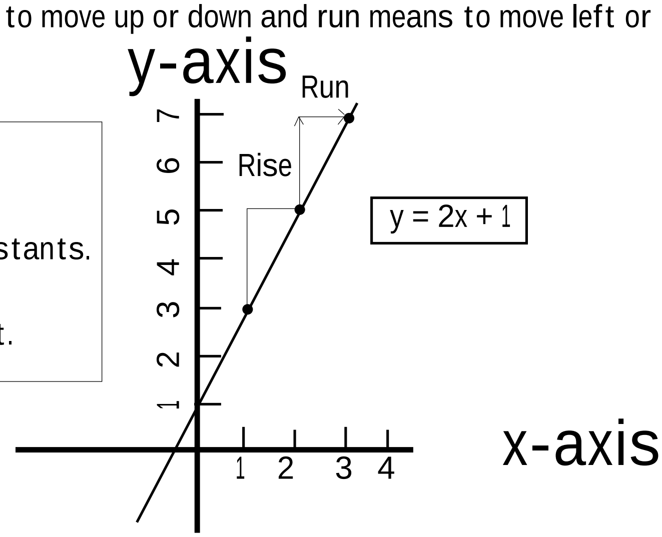
You made pictures of several different equations on the graph. Each equation had its own unique line with specific characteristics. However, All the lines were straight. Because all these equations form straight lines they are called LINE, or LINEAR EQUATIONS.

When an equation is written in this format, for example, $y = 2x + 1$, it is called the Y - INTERCEPT FORM of a LINEAR EQUATION. The Y- INTERCEPT form is important because you can very quickly tell the slant or slope of the line, and the point where the line crosses, or intercepts, the y axis. The number in front of the x tells the slope of the line, and the number following the + symbol tells the location of the y intercept. In the example, $y = 2x + 1$, the slope is 2 and the y intercept is 1. A slope of 2 means that for every two spaces up, move one space to the right. This is sometimes referred to as the RISE and RUN. Rise means to move up or down and run means to move left or right.

The general equation for the
Y-INTERCEPT FORM is

$y = mx + b$, where m and b are constants.

The constant m is the slope.
The constant b is the y - intercept.

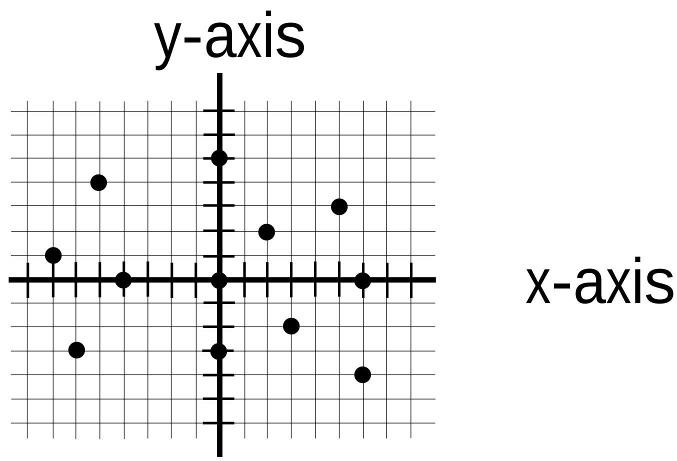


EXPANDING THE CONCEPT — PART A: Cartesian Coordinate System

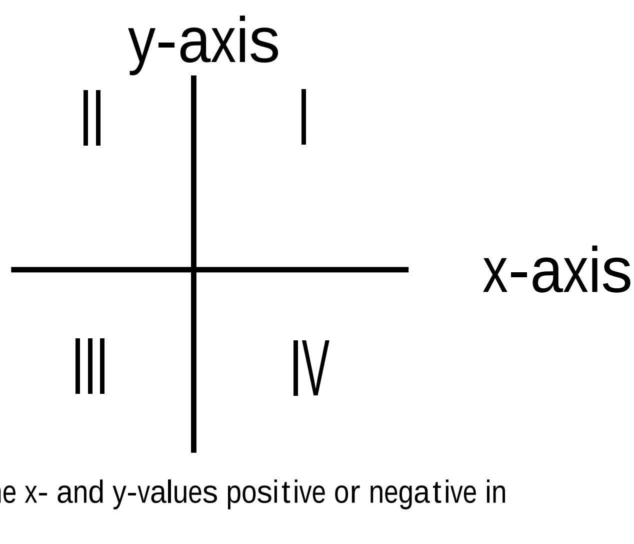
1. Use graph paper to plot the points of each ordered pair on the Cartesian coordinate system. The numbers right of the origin on the x-axis are positive. The numbers left of the origin on the x-axis are negative. The numbers above the origin on the y-axis are positive. The numbers below the origin on the y-axis are negative. Remember, in the ordered pair, the first number is the x-value, and the second number is the y-value. Be sure to label each point.

a. (2, 5)
b. (7, 3)
c. (-1, 3)
d. (3, - 5)
e. (-3, -6)

2. Write the coordinates of each point as an ordered pair.



You will notice that the Cartesian Coordinate system divides the graph paper into four regions. These regions are called QUADRANTS. The quadrants are numbered counterclockwise, starting at the upper right as shown below.



3. In the ordered pair, (x, y), are the x- and y-values positive or negative in

a. Quadrant I? X is __________; y is __________
b. Quadrant II? X is __________; y is __________
c. Quadrant III? X is __________; y is __________
d. Quadrant IV? X is __________; y is __________